Reply by Author to I. M. Grinberg and M. H. McLaughlin

GEORGE T. Y. CHAO* AND JOHN A. JACOBSEN†
Hercules Powder Company, Cumberland, Md.

AND

JAMES Z. ANDERSON‡
University of Nevada, Reno, Nev.

WE do not propose the use of the flat-plate model for the purpose of calculating nozzle temperature profiles as might be inferred from reading the poorly written second sentence of our Engineering Note.[1] The flat-plate model is used only to select the size of both the surface node thickness and associated time steps in order that space and time truncation errors may be reduced to as little as 1%. Use of the flat-plate model instead of a hollow cylinder for this purpose assumes that space truncation errors are independent of model geometry. This assumption is valid because the node can be taken as small enough to be described as a rectangular parallelepiped, regardless of the shape of the model. If this is true, using a hollow cylinder for a model would not produce a calibration that was any more accurate than the flat-plate calibration, but it would be considerably more difficult and time-consuming to accomplish. If this is not true, neither the flat-plate nor the hollow-cylinder model can be used to calibrate the numerical method to satisfy the complex geometry of the nozzle.

Thus, the purposes of our note were to 1) justify accuracy levels in finite-difference solutions to transient heat transfer in rocket nozzles and 2) give the reader some guidance as to the magnitudes of effort that would be required in either the simple approximate analytical solution, in terms of the eigenvalues necessary, or in the magnitudes of incremental slices and time steps necessary in the finite-difference approach. We did not intend to suggest that the flat-plate solution would be advisable for other than very approximate preliminary use in the analysis of rocket nozzles. Although the cylinder would be a step in refinement compared to the flat plate, the numerical solution to be used for an actual rocket nozzle would not be based either upon a flat plate, or a hollow cylinder, but would be based upon the actual geometric configuration of the rocket nozzle in question. The point is that, because rocket nozzles are complex (both in terms of geometry materials and boundary conditions), the numerical method can approximate the true conditions of a rocket nozzle much closer than any analytical method. Then, if the values of the input parameters are selected to minimize the various numerical errors (an outstanding one of which is truncation error), the numerical method is in an excellent position to properly duplicate actual rocket nozzle operating conditions.

Once the calibration for truncation errors has been performed, a set of curves, as shown in Figs. 4 and 5 of Ref 1, which guide the selection of surface node thickness (and also time steps) is drawn. From this time forward, all uses of the numerical method should be guided by these curves.

We might point out that there are analytical solutions for the hollow cylinder other than that of the reviewer (e.g., Ref. 2); in fact, handbooks of graphical solutions[3,4] thereof exist. A more complete version of our note appears in Ref. 5.

Received October 27, 1964; revision received December 31, 1964.

* Staff Engineer, Allegany Ballistics Laboratory. Associate Fellow Member AIAA.

† Development Mathematician, Allegany Ballistics Laboratory. Member AIAA.

‡ Dean, College of Engineering; formerly Consultant and Chairman, Department of Mechanical Engineering, West Virginia University, Morgantown, W. Va.

References

[1] Chao, G. T. Y., Jacobsen, J. A., and Anderson, J. T., "Transient surface temperatures in rocket nozzles," J. Spacecraft Rockets **1**, 219–221 (1964).

[2] Carslaw, H. S. and Jaeger, J. C., *Conduction of Heat in Solids* (Oxford University Press, Oxford, England, 1959), 2nd ed.

[3] Schneider, P. J., *Temperature Response Charts* (John Wiley and Sons, Inc., New York, 1963), Chart 36.

[4] Desmon, L. G. and Avis, G. B., "Transient body temperature response curves for hollow cylinders with heated interior and insulated exterior," *Cylheat Handbook*, Vol. I, Allegany Ballistics Lab. Rept. X-123 (July 1964).

[5] Chao, G. T. Y., Jacobsen, J. A., and Anderson, J. T., "Methods for calculation of inside wall surface transient temperatures for solid propellant rocket nozzles," Allegany Ballistics Lab. Rept. Z-73 (July 1964).

Addendum: "Application of Biot's Variational Method to Convective Heating of a Slab"

HUGH N. CHU*
North American Aviation, Inc., Canoga Park, Calif.

[J. Spacecraft Rockets **1**, 686–688 (1964)]

THE discussion toward the end of the original paper was based on the parabolic profile solution (3) as a standard of comparison primarily because it was first given by Biot (Ref. 1 of the original paper). Actually it is no better than, say, a cubic or quartic profile solution. Therefore, the concluding sentence of the paper is perhaps too restrictive. That is, within the spirit of approximation of the paper, Eq. (21) or a similar equation derivable from a quartic profile, say, should be applicable to the entire time range. On the other hand, strictly speaking, q_1 is not a good basis for comparison since the exact solution gives a penetration distance of infinity.

Received December 11, 1964.

* Senior Technical Specialist, Research Department, Rocketdyne Division. Member AIAA.